

11.5: 2, 20, 28, 34, 36

2. $z = x \ln(x + 2y), x = \sin t, y = \cos t \Rightarrow$

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = \left[x \cdot \frac{1}{x+2y} + 1 \cdot \ln(x+2y) \right] \cos t + x \cdot \frac{1}{x+2y} (2) \cdot (-\sin t) \\ &= \left[\frac{x}{x+2y} + \ln(x+2y) \right] \cos t - \frac{2x}{x+2y} (\sin t) \end{aligned}$$

20. $M = xe^{y-z^2}, x = 2uv, y = u - v, z = u + v \Rightarrow$

$$\begin{aligned} \frac{\partial M}{\partial u} &= \frac{\partial M}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial M}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial M}{\partial z} \frac{\partial z}{\partial u} = e^{y-z^2} (2v) + xe^{y-z^2} (1) + x(-2z)e^{y-z^2} (1) \\ &= e^{y-z^2} (2v + x - 2xz), \end{aligned}$$

$$\begin{aligned} \frac{\partial M}{\partial v} &= \frac{\partial M}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial M}{\partial y} \frac{\partial y}{\partial v} + \frac{\partial M}{\partial z} \frac{\partial z}{\partial v} = e^{y-z^2} (2u) + xe^{y-z^2} (-1) + x(-2z)e^{y-z^2} (1) \\ &= e^{y-z^2} (2u - x - 2xz). \end{aligned}$$

When $u = 3, v = -1$ we have $x = -6, y = 4,$ and $z = 2,$ so $\frac{\partial M}{\partial u} = 16$ and $\frac{\partial M}{\partial v} = 36.$

28. $yz = \ln(x+z),$ so let $F(x, y, z) = yz - \ln(x+z) = 0.$ Then $\frac{\partial z}{\partial x} = \frac{F_x}{F_z} = \frac{-\frac{1}{x+z}(1)}{y - \frac{1}{x+z}(1)} = \frac{1}{y(x+z) - 1},$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z}{y - \frac{1}{x+z}} = -\frac{z(x+z)}{y(x+z) - 1}.$$

34. $I = \frac{V}{R} \Rightarrow \frac{dI}{dt} = \frac{\partial I}{\partial V} \frac{dV}{dt} + \frac{\partial I}{\partial R} \frac{dR}{dt} = \frac{1}{R} \frac{dV}{dt} - \frac{V}{R^2} \frac{dR}{dt} = \frac{1}{R} \frac{dV}{dt} - \frac{I}{R} \frac{dR}{dt}$

$$= \frac{1}{400}(-0.01) - \frac{0.08}{400}(0.03) = -0.000031 \text{ A/s}$$

36. $f_o = \left(\frac{c + v_o}{c - v_s} \right) f_s = \left(\frac{332+34}{332-40} \right) 460 \approx 576.6 \text{ Hz.}$ v_o and v_s are functions of time $t,$ so

$$\begin{aligned} \frac{df_o}{dt} &= \frac{\partial f_o}{\partial v_o} \frac{dv_o}{dt} + \frac{\partial f_o}{\partial v_s} \frac{dv_s}{dt} = \left(\frac{1}{c - v_s} \right) f_s \cdot \frac{dv_o}{dt} + \frac{c + v_o}{(c - v_s)^2} f_s \cdot \frac{dv_s}{dt} \\ &= \left(\frac{1}{332-40} \right) (460) (1.2) + \frac{332+34}{(332-40)^2} (460) (1.4) \approx 4.65 \text{ Hz/s} \end{aligned}$$